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Signals of universality violation in e^+e^- collisions

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Abstract. We examine distributions of leptons produced in e^+e^- collisions, by a family nonuniversal extra gauge boson Z', suggested by the $\mathrm{Sp}(6)_\mathrm{L}\otimes\mathrm{U}(1)_Y$ model, and by other neutral gauge bosons occurring in left–right symmetric models and in superstring-inspired E_6 models. We discuss how to distinguish the models by examining the couplings to fermions of the extra Z-boson through its leptonic production cross sections and asymmetries. We show how the universality violation inherent in the $\mathrm{Sp}(6)_\mathrm{L}\otimes\mathrm{U}(1)_Y$ model enhances the discovery potential of Z' at future planned and proposed e^+e^- colliders.

The family replication of quarks and leptons is a well established phenomenon. However, this phenomenon is not well understood. In fact, as far as the standard model (SM) [1, 2] is concerned, the generation universality is simply put in by hand. Although generation universality has experimentally been realized for the first two generations, the possibility of a small universality violation for the third generation is still viable. Recent phenomenological analyses [3, 4] have indeed concluded that extra neutral gauge bosons with a TeV scale mass and family nonuniversal couplings are severely constrained by experimental results on flavor changing processes that involve couplings of an extra Z to first and second generation fermions. However, couplings to the third generation are found to be much less constrained. The origin of universality violation by the third generation could arise from a number of sources. The $\mathrm{Sp}(6)_\mathrm{L}\otimes\mathrm{U}(1)_Y$ family model [5–7], which naturally accommodates such violations, offers one possibility. The $\mathrm{Sp}(6)_\mathrm{L}\otimes\mathrm{U}(1)_Y$ model predicts the existence of a set of intergenerational, horizontal gauge bosons, keeping the fermion spectrum intact. In the $\mathrm{Sp}(6)_\mathrm{L}\otimes\mathrm{U}(1)_Y$ model, the standard $\mathrm{SU}(2)_\mathrm{L}$ is unified with the horizontal gauge group $G_\mathrm{H}(=\mathrm{SU}(3)_\mathrm{H})$ into an anomaly free, simple, Lie group. The six left-handed quarks (or leptons) belong to a **6** of $\mathrm{Sp}(6)_\mathrm{L}$, while the right-handed fermions are all singlets. It is thus a straightforward generalization of $\mathrm{SU}(2)_\mathrm{L}$ into $\mathrm{Sp}(6)_\mathrm{L}$, with the three doublets of $\mathrm{SU}(2)_\mathrm{L}$ coalescing into a sextet of $\mathrm{Sp}(6)_\mathrm{L}$. Sp(6) can be naturally broken into $[\mathrm{SU}(2)]^3=\mathrm{SU}(2)_1\otimes\mathrm{SU}(2)_2\otimes\mathrm{SU}(2)_3$, where $\mathrm{SU}(2)_i$ operates on the ith generation exclusively. Thus the standard $\mathrm{SU}(2)_\mathrm{L}$ is to be identified with the diagonal SU(2) subgroup of $[\mathrm{SU}(2)]^3$. In terms of the $\mathrm{SU}(2)_i$ gauge boson, $\mathbf{A}_i$, the $\mathrm{SU}(2)_\mathrm{L}$ gauge bosons are given by $\mathbf{A}=(1/\sqrt{3})(\mathbf{A}_1+\mathbf{A}_2+\mathbf{A}_3)$. Of the other orthogonal combinations of $\mathbf{A}_i$, $\mathbf{A}'=(1/\sqrt{6})(\mathbf{A}_1+\mathbf{A}_2-2\mathbf{A}_3)$, which exhibits universality only among the first two generations, can have a mass scale in the TeV range [8]. The additional gauge bosons $\mathbf{A}'$, denoted by Z' and $W'^{\pm}$, suggest new physics [9–26] beyond the standard model.

In the $\mathrm{Sp}(6)_\mathrm{L}\otimes\mathrm{U}(1)_Y$ model the neutral gauge boson Z' couples equally to the first and second generation, but differently to the third. This universality violation gives rise to distinctive observable features. In this work we wish to examine and compare characteristic signals in e^+e^- collisions, resulting from the presence of Z', with those from other theoretically motivated neutral gauge bosons. In particular, we will consider here the neutral gauge bosons, Z_LR occurring in left–right symmetric models (LR), Z_χ, Z_ψ, Z_η and $Z_{I'}$ occurring in grand unified theories (GUTs) based on E_6 and SO(10) groups (including superstring models) [27–31]. We would like also to study the discovery potential of Z' in future e^+e^- colliders.

The presence of the additional gauge boson modifies the neutral-current Lagrangian so as to contain an additional term

$$-\mathcal{L}_\mathrm{N.C.}=eJ^\mu_\mathrm{em}A_\mu+g_1J_1^\mu Z^0_{1\mu}+g_2J_2^\mu Z^0_{2\mu}\,, \tag{1}$$

where Z_1^0 is the $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_Y$ boson and Z_2^0 is the additional boson in the weak eigenstate basis. The g_i, $i=1,2$, are the gauge couplings with $g_1=g/\cos\vartheta_\mathrm{W}$, where $g=e/\sin\vartheta_\mathrm{W}$. For the $\mathrm{Sp}(6)_\mathrm{L}\otimes\mathrm{U}(1)_Y$ model $g_2=\sqrt{(1-x)/2}\,g_1=g/\sqrt{2}$ and $x=\sin^2\vartheta_\mathrm{W}$. The neutral currents J_i, $i=1,2$, are given by

$$J_i^\mu=\frac{1}{2}\sum_f\overline{\psi}_f\gamma^\mu\left[g_V^{(i)}(f)+g_A^{(i)}(f)\gamma_5\right]\psi_f\,. \tag{2}$$

Here $g^{(i)}_{\mathrm{V,A}}(f)$ are the vector and axial-vector couplings of the fermion f to Z_i^0, respectively. After symmetry breaking the weak eigenstate bosons Z_i^0 are related to the mass

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eigenstate bosons Z_i by

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} Z_1^0 \\ Z_2^0 \end{pmatrix}, \quad (3)$$

where φ denotes the mixing angle between Z_1^0 and Z_2^0. The neutral-current Lagrangian now reads

$$-\mathcal{L}_{\text{N.C.}} = \quad g_1 \sum_{i=1}^{2} \left[\sum_f \overline{\psi}_f \gamma_\mu \left(v_i^f + a_i^f \gamma_5 \right) \psi_f \right] Z_i^\mu, \quad (4)$$

where

$$v_1^f, a_1^f = \frac{1}{2} \left[g_{\text{V,A}}^{(1)}(f) \cos\varphi + \frac{g_2}{g_1} g_{\text{V,A}}^{(2)}(f) \sin\varphi \right], \quad (5)$$

$$v_2^f, a_2^f = \frac{1}{2} \left[-g_{\text{V,A}}^{(1)}(f) \sin\varphi + \frac{g_2}{g_1} g_{\text{V,A}}^{(2)}(f) \cos\varphi \right]. \quad (6)$$

For the SM $g_{\text{V}}^{(1)}(f) = (T_{3\text{L}} - 2xQ)_f$ and $g_{\text{A}}^{(1)}(f) = (T_{3\text{L}})_f$. Here $(T_{3\text{L}})_f$ and Q_f are the third component of weak isospin and electric charge of fermion f, respectively. For the $\text{Sp}(6)_\text{L} \otimes \text{U}(1)_Y$ model $g_{\text{V}}^{(2)}(f) = g_{\text{A}}^{(2)}(f) = (T_{3\text{L}})_f$, for the first two generations and $g_{\text{V}}^{(2)}(f) = g_{\text{A}}^{(2)}(f) = -2(T_{3\text{L}})_f$ for the third one. For the GUT cases considered here, the couplings are given in [27–30].

Several articles have dealt with phenomenological effects resulting from the presence of theoretically motivated, additional neutral gauge bosons [32–54]. However, to date, there is no experimental evidence from the Fermilab Tevatron for the existence of any additional neutral gauge bosons [55]. To push Z' searches into the TeV region will require e^+e^- colliders with energies in the range 500 GeV–1 TeV or new hadron colliders such as the large hadron collider (LHC) at CERN. The LHC, due for completion in 2008, will accelerate protons to achieve center of mass energies, $\sqrt{s} = 14$ TeV. At energies of this magnitude additional neutral gauge bosons could be observed at the LHC. However, in pp (or $p\overline{p}$) collisions, the hard collision between two constituents occur at (unknown) energy typically $\lesssim \frac{1}{6}$ of the available energy. Moreover, hadronic interactions are associated with large background processes. The discovery limits for Z' in hadronic collisions were studied elsewhere [56–60]. The e^+e^- colliders have the advantage over hadron colliders of a clean environment for the production of fermion and anti-fermion pairs. Moreover, in contrast with hadronic collisions, all the center of mass energy is available for the production of these particles. The need for a low background high-energy collider is being addressed by the proposed e^+e^- international linear collider (ILC) [61], which is expected to operate in 2015. The collider will be capable of reaching center of mass energies in the range 500 GeV–1 TeV with an annual luminosity in the range $L = 0.5\,\text{ab}^{-1}$ to $L = 1\,\text{ab}^{-1}$. The expected energy reach and annual luminosity for the first stage of the ILC are $\sqrt{s} = 500$ GeV and $L = 0.5\,\text{ab}^{-1}$. The e^+e^- collision energy frontier next to ILC is expected to be provided by the compact linear collider (CLIC), which is designed to collide electrons and positrons at center of mass energies from 1 TeV to 5 TeV [62]. The CLIC design parameters have been optimized for nominal center of mass energy $\sqrt{s} = 3$ TeV with annual luminosity of about $1\,\text{ab}^{-1}$. The clean nature and high energy expected at the e^+e^- colliders would certainly enable measurements of the properties of extra gauge bosons with precision. We will be interested here in studying characteristic signals in e^+e^- collisions resulting from the presence of the additional neutral gauge bosons. The neutral-current Lagrangian in the mass eigenstate basis can be used to determine the cross section $\sigma(e^+e^- \to f\overline{f})$. The integrated forward–backward asymmetry is defined as

$$A_{\text{FB}} = \frac{\int_0^1 (\text{d}\sigma/\text{d}z)\,\text{d}z - \int_{-1}^0 (\text{d}\sigma/\text{d}z)\,\text{d}z}{\int_{-1}^1 (\text{d}\sigma/\text{d}z)\,\text{d}z}, \quad (7)$$

where $z = \cos\vartheta$ and ϑ is the angle between the outgoing fermion and the incident electron. On the other hand, the left–right asymmetry is defined as

$$A_{\text{LR}} = \frac{\sigma_\text{L} - \sigma_\text{R}}{\sigma_\text{L} + \sigma_\text{R}}, \quad (8)$$

where σ_L (σ_R) is the cross section for a left- (right-) handed electron on an unpolarized positron. For electron polarization less than 100% the asymmetry is given by $A_{\text{LR}}^P = PA_{\text{LR}}$, where P is the longitudinal degree of polarization. With the couplings given by (5) and (6), the general expressions for the forward–backward and left–right asymmetries are written explicitly as

$$A_{\text{FB}} = \frac{3}{4}\frac{F_2}{F_1}, \quad A_{\text{LR}} = \frac{F_3}{F_1}, \quad (9)$$

where

$$F_1 = 1 + 2\sum_{j=1}^{2} v_j^e v_j^f \chi_j + \sum_{j,k=1}^{2} (\chi_j\chi_k + \eta_j\eta_k) \left(v_j^e v_k^e + a_j^e a_k^e\right) \left(v_j^f v_k^f + a_j^f a_k^f\right), \quad (10)$$

$$F_2 = 2\sum_{j=1}^{2} a_j^e a_j^f \chi_j + \sum_{j,k=1}^{2} (\chi_j\chi_k + \eta_j\eta_k) \left(v_j^e a_k^e + a_j^e v_k^e\right) \left(v_j^f a_k^f + a_j^f v_k^f\right), \quad (11)$$

$$F_3 = 2\sum_{j=1}^{2} v_j^e a_j^f \chi_j + 2\sum_{j,k=1}^{2} (\chi_j\chi_k + \eta_j\eta_k)\, v_j^e a_k^e \left(v_j^f v_k^f + a_j^f a_k^f\right), \quad (12)$$

and

$$\chi_j = \frac{s\left(s - M_j{}^2\right)}{x(1-x)\left[\left(s - M_j{}^2\right)^2 + M_j{}^2 \Gamma_j{}^2\right]}\,, \qquad (13)$$

$$\eta_j = \frac{-sM_j\Gamma_j}{x(1-x)\left[\left(s - M_j{}^2\right)^2 + M_j{}^2 \Gamma_j{}^2\right]}\,. \qquad (14)$$

Here M_j and Γ_j are the mass and total width of gauge boson Z_j, respectively. The Z_2 total width [63] is given by

$$\begin{aligned}\Gamma_2 &= \sum_f \Gamma\left(Z_2 \to \overline{f} f\right)\\ &= \sum_f N_f \mu M_2 \frac{\sqrt{2} G_{\mathrm{F}} M_1^2}{3\pi}\left(1 + \frac{3\alpha}{4\pi} Q_f^2\right) R_{\mathrm{QCD}}\\ &\quad\times\left\{\left(\left(v_2^f\right)^2 + \left(a_2^f\right)^2\right)\left(1 + \frac{2m_f^2}{M_2^2}\right) - 6\left(a_2^f\right)^2 \frac{m_f^2}{M_2^2}\right\}.\end{aligned} \qquad (15)$$

N_f is the color factor where $N_f = 1$ for leptons and $N_f = 3$ for quarks. μ is the phase space factor due to the massive final fermion, $\mu = \sqrt{1 - 4m_f^2/M_2^2}$. $R_{\mathrm{QCD}} = 1$ for leptons, and for quarks

$$\begin{aligned}R_{\mathrm{QCD}} = 1 &+ \frac{\alpha_{\mathrm{s}}\left(M_2^2\right)}{\pi} + 1.405\frac{\alpha_{\mathrm{s}}^2\left(M_2^2\right)}{\pi^2}\\ &- 12.8\frac{\alpha_{\mathrm{s}}^3\left(M_2^2\right)}{\pi^3} - \left(\frac{\left(Q_f\right)^2}{4}\right)\frac{\alpha\alpha_{\mathrm{s}}\left(M_2^2\right)}{\pi^2}\,.\end{aligned} \qquad (16)$$

The universality violation inherent in the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model is expected to play an important role in distinguishing the model and in enhancing the discovery limits for Z' in future planned and proposed e^+e^- colliders. In what follows we will attempt to verify the above statement by examining the observables $\sigma(l) \equiv \sigma(e^+e^- \to l^+l^-)$, $A_{\mathrm{FB}}(l) \equiv A_{\mathrm{FB}}(e^+e^- \to \bar{l}l)$ and $A^P_{\mathrm{LR}}(l) \equiv A^P_{\mathrm{LR}}(e^+e^- \to \bar{l}l)$ for $l = \mu$ and τ, as predicted by the models considered here. The deviations of these observables from the SM values depend on the Z_2 couplings to leptons. The dominant contribution to $\sigma(l)$, on and within few total widths of the Z_2 resonance peak is proportional to the ratio, $r = C(l)/(\Gamma_2)^2$, where $C(l) = [(v_2^e)^2 + (a_2^e)^2][(v_2^l)^2 + (a_2^l)^2]$. However, the effect of the total width on the measurable observables can be neglected several widths away from the extra gauge boson resonance. Furthermore, measurements at one energy point sufficiently far away from the Z_1 and Z_2 resonances can only restrict the normalized couplings $(v_2^l, a_2^l)^N = (v_2^l, a_2^l)\sqrt{g_1^2 \frac{s}{M_2^2 - s}}$. In our analysis, we make the simplifying assumption that the mixing angle, φ, can be ignored, as it is constrained to be tiny for all the models considered in this work [64–67]. In calculating total widths for the GUTS models considered here, we use assumptions identical to those employed in [68]. We calculated the cross sections $\sigma(\mu)$ and $\sigma(\tau)$, as predicted by the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model, where we assumed that the gauge boson mass, $M_2 \equiv M = 1$ TeV and $\varphi = 0$. In fact, recent direct- and indirect-search lower bounds on M, for any of the extra gauge bosons considered here, showed that $M \gtrsim 0.5$ TeV [69–73]. The cross sections are presented in Fig. 1 as functions of $\sqrt{s}$. An integrated luminosity of $L = 1.0\,\mathrm{ab}^{-1}$ per 10^7 sec year of running is expected to be achieved in the e^+e^- linear collider operating at $\sqrt{s} = 1$ TeV, [74, 75]. For $M = 1$ TeV, a run on-resonance with the given integrated luminosity would yield approximately 7×10^6 Z' events decaying into a $\mu^+\mu^-$ pair and 2.8×10^7 Z' events decaying into a $\tau^+\tau^-$ pair a year. This four-fold enhancement in the $\tau^+\tau^-$ production rate relative to the $\mu^+\mu^-$ production rate is provided by the generation-dependent couplings of Z' to leptons and can be used to distinguish the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model. Of the other models considered in this analysis, we will only be concerned here with those models that make predictions that can be confused with those of the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model. Figure 1 shows that the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model predictions for the muon-pair production cross section, $\sigma_{\mathrm{Sp}}(\mu)$, overlap those of model I', $\sigma_{I'}(\mu)$, except only on and near the Z_2 resonance peak.[1] This can be explained as follows: for $l = \mu$, the factor $C(l)$ is identical in both models where for the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model, $(g_2/g_1)^2_{\mathrm{Sp}} = \cos^2\vartheta_{\mathrm{W}}/2 \approx 0.38$, and for model I', $(g_2/g_1)^2_{I'} = (5/3)x \approx 0.38$, and for both models, $(g^{(2)}_{\mathrm{V,A}}(e,\mu))^2 = 0.25$. Therefore on top of the Z_2 resonance, the ratio of the peak values is $[\sigma_{I'}(\mu)]_p/[\sigma_{\mathrm{Sp}}(\mu)]_p \propto (\Gamma_{Z'}/\Gamma_{Z_{I'}})^2 \sim 1.72$. Away from the resonance peak, the $\sigma(\mu)$ predictions of these two models are identical because the magnitudes of the normalized couplings are. Figure 1 shows also that on top of the Z_2 resonance, $[\sigma_{\mathrm{Sp}}(\tau)]_p \approx [\sigma_\chi(\tau)]_p$, which is a result of $r_{\mathrm{Sp}} \approx r_\chi$. Here, neither the values of $C(\tau)$ nor those of Γ_2 are equal in these two models; however, $C_\chi(\tau)/C_{\mathrm{Sp}}(\tau) \approx (\Gamma_{Z_\chi}/\Gamma_{Z'})^2$. On the other hand, because the normalized couplings of Z_2 to the tau leptons are different in these two models, they gave different predictions for $\sigma(\tau)$ away from the Z_2 resonance. The deviations of the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model predictions from the SM predictions at $\sqrt{s} = 0.5$ TeV are $\sim 10.5\%$ for $\sigma(\mu)$ and $\sim 25.3\%$ for $\sigma(\tau)$.

We would like next to examine the leptonic forward–backward and left–right asymmetries, as predicted by the models considered in this work. In Figs. 2 and 3 we present, respectively, $A_{\mathrm{FB}}(\mu)$ and $A^P_{\mathrm{LR}}(\mu)$ as functions of $\sqrt{s}$ for all models, including the standard model. Also shown are $A_{\mathrm{FB}}(\tau)$ and $A^P_{\mathrm{LR}}(\tau)$ as predicted by the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model. We take the gauge boson mass, $M = 1$ TeV and assume no mixing, $\varphi = 0$. The expressions for $A_{\mathrm{FB}}(l)$ and $A^P_{\mathrm{LR}}(l)$ depend on the quantities F_1, F_2 and F_3. Equations (10)–(12) show that the expressions for F_1, F_2 and F_3 contain only products of an even number of couplings of Z_2 to fermions. Therefore the process $e^+e^- \to \mu^+\mu^-$ cannot distinguish between models which differ only by the signs of all Z_2 couplings to fermions. Figure 2 shows that the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model predictions for $A_{\mathrm{FB}}(\mu)$ are identical to those of model I'. This is expected since the vector and

[1] We note that the gauge boson $Z_{I'}$ associated with model I' considered here is a linear combination of Z_ψ and Z_χ [27–30].

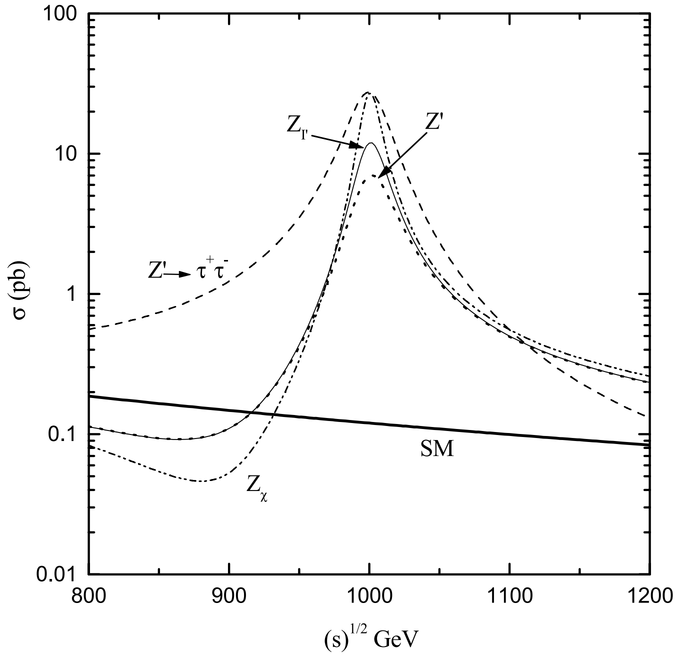

Fig. 1. The cross sections $\sigma(\mu)$, as function of $\sqrt{s}$, for the $\mathrm{Sp(6)_L \otimes U(1)}_Y$ model (*dotted curve*), model I' (*solid curve*) and the standard model (*bold solid curve*). Also shown are the cross sections $\sigma(\tau)$ for the $\mathrm{Sp(6)_L \otimes U(1)}_Y$ model (*dashed curve*) and model χ (*dash double-dotted curve*). We take $M = 1$ TeV and $\varphi = 0$

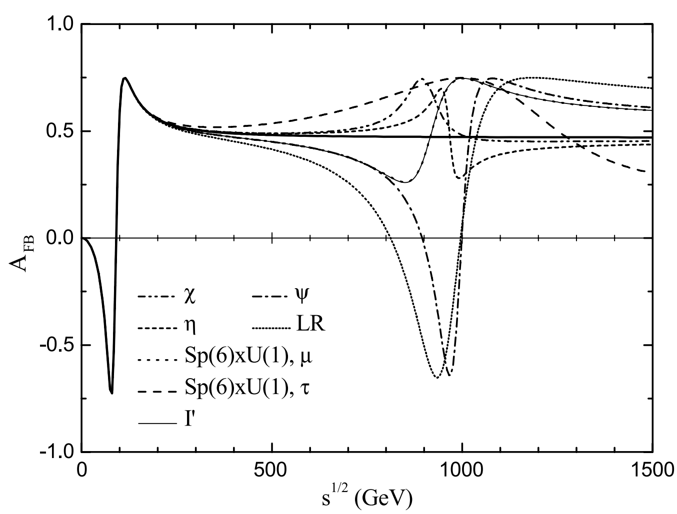

Fig. 2. The forward–backward asymmetries, $A_{\mathrm{FB}}(\mu)$, as functions of $\sqrt{s}$, for the $\mathrm{Sp(6)_L \otimes U(1)}_Y$ model, the left–right symmetric model and string models, with $M = 1$ TeV and $\varphi = 0$. The *bold solid curve* is the standard model predictions. Also shown is $A_{\mathrm{FB}}(\tau)$ for the $\mathrm{Sp(6)_L \otimes U(1)}_Y$ model

axial-vector couplings of e and μ leptons to Z_2 in these two models differ only by a sign. The effect of the total width at the Z_2 resonance did not show up because, on resonance, Γ_2 cancels in the ratio F_2/F_1 (see (9)). It will therefore be difficult to distinguish these two models through only measurements of $A_{\mathrm{FB}}(\mu)$. However, the $\mathrm{Sp(6)_L \otimes U(1)}_Y$ model predictions for $A_{\mathrm{FB}}(\tau)$ in the energy range $0.4M \lesssim \sqrt{s} \lesssim 0.8M$, are well separated from any of the other models. It will therefore be necessary to measure $A_{\mathrm{FB}}(\tau)$ in this energy region, in order to distinguish the $\mathrm{Sp(6)_L \otimes U(1)}_Y$ model through leptonic forward–backward asymmetries. The deviations of the $\mathrm{Sp(6)_L \otimes U(1)}_Y$ model from the SM predictions at $\sqrt{s} = 0.5$ TeV, are $\sim 6.5\%$ for $A_{\mathrm{FB}}(\mu)$ and $\sim 11\%$ for $A_{\mathrm{FB}}(\tau)$. In Fig. 3 we present the left–right asymmetries for all models, as functions of $\sqrt{s}$, where we take $M = 1$ TeV and $\varphi = 0$. An electron polarization of $\gtrsim 80\%$ is expected to be achieved both in the ILC and CLIC [76]. We therefore use $P = 80\%$ in our calculations. Except for model ψ, the deviations from the SM predictions are dramatic. Figure 3 shows that the predictions of the $\mathrm{Sp(6)_L \otimes U(1)}_Y$ model for $A^P_{\mathrm{LR}}(\mu)$ overlap those for model I' as expected. However, in the energy range $0.25M \lesssim \sqrt{s} \lesssim M$, the $\mathrm{Sp(6)_L \otimes U(1)}_Y$ model predictions for $A^P_{\mathrm{LR}}(\tau)$ are well separated from the corresponding predictions of any of the other models. At $\sqrt{s} = 0.5$ TeV, the deviations of the $\mathrm{Sp(6)_L \otimes U(1)}_Y$ model from the SM predictions are $\sim 165\%$ for $A^P_{\mathrm{LR}}(\mu)$ and $\sim 285\%$ for $A^P_{\mathrm{LR}}(\tau)$.

We have seen that, as for the $\mathrm{Sp(6)_L \otimes U(1)}_Y$ model, the deviations from the SM predictions are most pronounced for the observables $\sigma(\tau)$, $A_{\mathrm{FB}}(\tau)$ and $A^P_{\mathrm{LR}}(\tau)$. These large deviations are due to the universality violation inherent in the $\mathrm{Sp(6)_L \otimes U(1)}_Y$ model and are expected to enhance the discovery limits for Z' in e^+e^- collisions. We will thus study the discovery potential of Z' in e^+e^- collisions, by looking for significant deviations of observables from the SM predictions [77–84]. The predictions for the independent observables $\sigma(l)$, $A_{\mathrm{FB}}(l)$ and $A^P_{\mathrm{LR}}(l)$, where $l = \mu$ are calculated in e^+e^- collisions at $\sqrt{s} = 0.5$ TeV for all the models considered here. The results are shown in Fig. 4 as functions of the extra gauge boson mass, which is taken as a free parameter. The same observables, but with $l = \tau$, are also calculated and shown, but for the $\mathrm{Sp(6)_L \otimes U(1)}_Y$

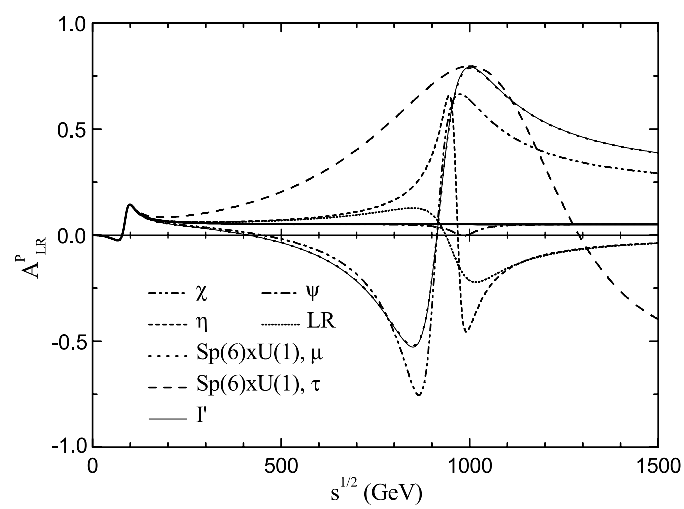

Fig. 3. The left–right asymmetries, $A^P_{\mathrm{LR}}(\mu)$, as functions of $\sqrt{s}$, for the $\mathrm{Sp(6)_L \otimes U(1)}_Y$ model, the left–right symmetric model and string models, with $M = 1$ TeV, $\varphi = 0$ and $P = 80\%$. The *bold solid curve* is the standard model predictions. Also shown is $A^P_{\mathrm{LR}}(\tau)$ for the $\mathrm{Sp(6)_L \otimes U(1)}_Y$ model

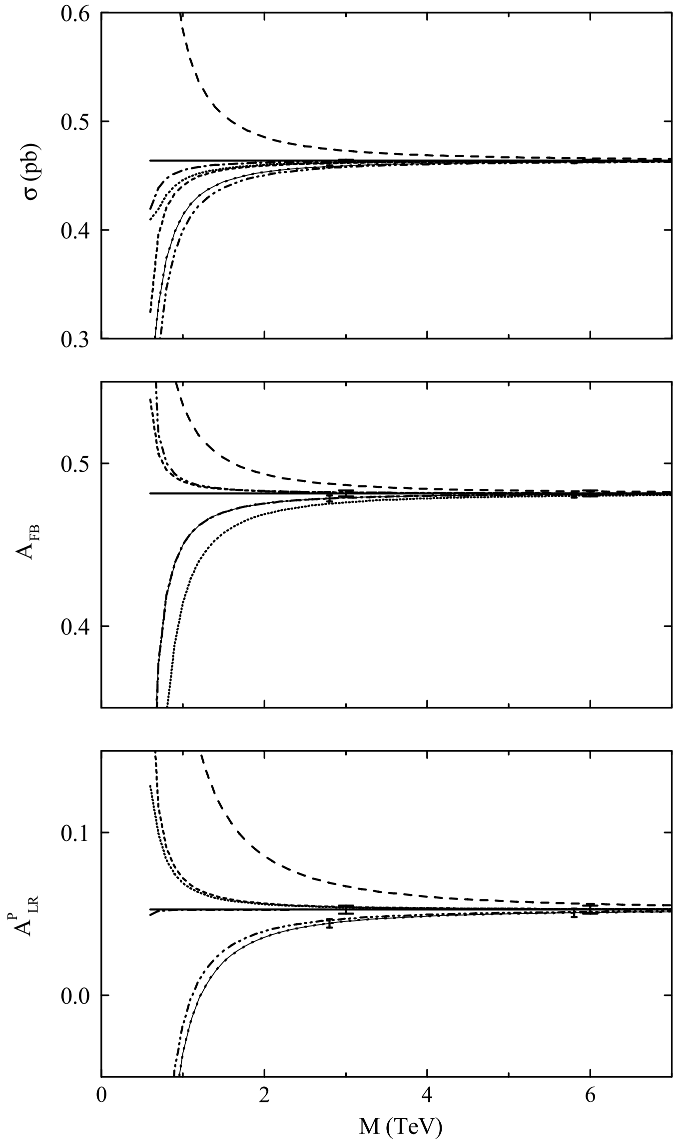

Fig. 4. The e^+e^- observables $\sigma(l), A_{\mathrm{FB}}(l)$ and $A_{\mathrm{LR}}^P(l)$ with $l = \mu$ and $P = 80\%$, as predicted by the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model, the left–right symmetric model and string models, as functions of M for $\sqrt{s} = 500$ GeV. Also shown are the same observables but with $l = \tau$, as predicted by the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model. The *bold solid lines* are the standard model values. Also shown are the error bars for the μ observables, as predicted by the SM (*bars with wide caps*) and by the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model (*bars with narrow caps*). The error bars are based on the statistical errors, assuming an integrated luminosity of $0.5\,\mathrm{ab}^{-1}$. The curve notations are as of Fig. 2

model. Also shown in Fig. 4 are the standard model predictions for the same observables along with their experimental errors. One notices that different observables have different sensitivities to the different models. As for the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model, one can easily notice the large deviations of observables having τ pairs in the final state. We obtain discovery limits for Z' by comparing the predictions for the observables considered here, assuming the presence of Z', to the predictions of the SM and constructing the χ^2 figure of merit:

$$\chi^2 = \sum_i \left(\frac{O_i^{Z'} - O_i^{\mathrm{SM}}}{\delta O_i^{\mathrm{SM}}} \right)^2 . \tag{17}$$

Here δO_i is the statistical error of the observable O_i. The contributions of the various observables to χ^2 for Z' are shown in Fig. 5. These are based on $\sqrt{s} = 0.5\,\mathrm{TeV}$, $L = 0.5\,\mathrm{ab}^{-1}$, $M_{Z'} = 2$ TeV. Figure 5 shows that the largest contributions to χ^2 come from the τ observables, $\sigma(\tau)$, $A_{\mathrm{FB}}(\tau)$ and $A_{\mathrm{LR}}^P(\tau)$. We therefore expect that these τ observables should play an important role in enhancing the Z' discovery limits. We calculated the 99% C.L. discovery limits for Z', in future planned and proposed e^+e^- colliders, according to (17), where we considered only the μ observables $\sigma(\mu)$, $A_{\mathrm{FB}}(\mu)$ and $A_{\mathrm{LR}}^P(\mu)$. We repeated the calculations but after including the corresponding τ observables. Figure 6 shows that the inclusion of the τ observables increased the discovery limits. For example, at $\sqrt{s} = 0.5$ TeV and $L = 0.5\,\mathrm{ab}^{-1}$, the limits jumped from ~ 4.4 TeV to ~ 6.56 TeV, a $\sim 50\%$ increase. With the τ observables included the limits range from ~ 6.5 TeV for $\sqrt{s} = 0.5$ TeV with $L = 0.5\,\mathrm{ab}^{-1}$ to ~ 19.5 TeV for $\sqrt{s} = 3$ TeV with $L = 1\,\mathrm{ab}^{-1}$. What we have done here is that we calculated, at specific values of $\sqrt{s}$ and L, the Z' mass up to which the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model can statistically be distinguished from the SM. It is necessary, however, that we, simultaneously, be able to distinguish the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model from the other models. We have seen the significance of the role of τ observables in this respect. It is therefore important to answer the following question: within the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model, up to what Z' mass does one see statistically relevant differences in μ versus τ observables? To answer this question, we construct the χ^2 figure of merit:

$$\chi^2_{\mu-\tau} = \sum_i \left(\frac{O_i^{Z'}(\tau) - O_i^{Z'}(\mu)}{\delta O_i^{Z'}(\mu)} \right)^2 . \tag{18}$$

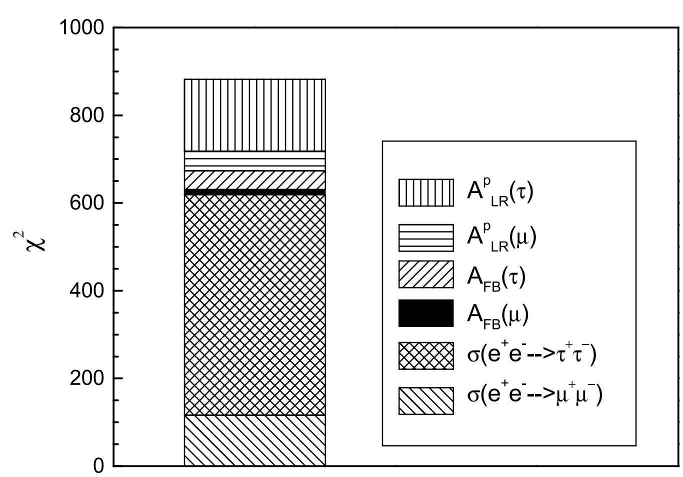

Fig. 5. The contributions to χ^2 for the observables $\sigma(l)$, $A_{\mathrm{FB}}(l)$ and $A_{\mathrm{LR}}^P(l)$ with $l = \mu$ and τ and $P = 80\%$, for the $\mathrm{Sp}(6)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ model. These are based on $\sqrt{s} = 0.5$ TeV, $L = 0.5\,\mathrm{ab}^{-1}$ and $M_{Z'} = 2$ TeV. The χ^2 is based only on the statistical error

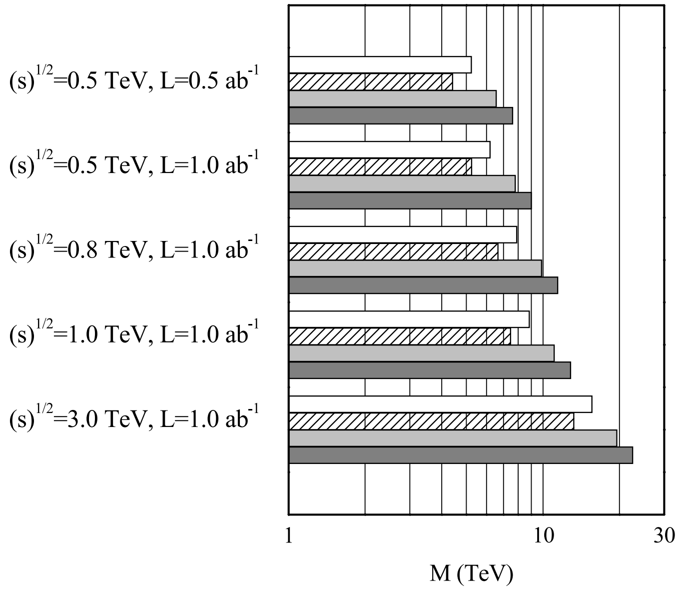

Fig. 6. Discovery limits for Z' (*shaded bars*), and $Z_{I'}$ (*open bars*), and the $\mathrm{Sp}(6)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ model μ–τ distinguishability limits (*solid bars*), at high energy e^+e^- colliders. The limits are 99% C.L. obtained from a χ^2 based on the observables $\sigma(l), A_{\mathrm{FB}}(l)$ and $A^P_{\mathrm{LR}}(l)$ with $l = \mu$ and τ and $P = 80\%$. Also shown are the 99% C.L. discovery limits for Z', based only on the $l = \mu$ observables (*hatched bars*). The integrated luminosities are based on a 10^7 sec year of running

Here $O_i^{Z'}(\mu)$ and $O_i^{Z'}(\tau)$ are observables having μ pairs and τ pairs in their final states, respectively. $\delta O_i^{Z'}(\mu)$ is the statistical error associated with the observable $O_i^{Z'}(\mu)$. We calculated $\chi^2_{\mu-\tau}$, for the same $\sqrt{s}$ and L values considered above. The solid bars in Fig. 6 represent what we call the μ–τ distinguishability limits, or the 99% C.L. for Z' masses, up to which the τ observables can statistically be distinguished from the μ observables. We stress here that these limits are obtained according to (18), which assumes the presence of no other models. In general, the μ–τ distinguishability limit, $M_{\mu-\tau}$, of a model having an additional gauge boson that couples equally to the first two generations, but differently to the third, can have any value larger or smaller than the discovery limit, M_d. If $M_{\mu-\tau} < M_\mathrm{d}$, it will not be possible to identify the originating model of a gauge boson discovered with a mass somewhere between $M_{\mu-\tau}$ and M_d. However, it will be possible to identify the original model only up to gauge boson masses satisfying $M \leq M_{\mu-\tau}$. That is to say, the discovery (with identification) limit is forced down to the μ–τ distinguishability limit. In the opposite case, that is, when $M_{\mu-\tau} > M_\mathrm{d}$, the μ–τ distinguishability limit is forced down to the discovery limit. In the special case of the $\mathrm{Sp}(6)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ model, $M_{\mu-\tau} > M_\mathrm{d}$ (see Fig. 6). This result is expected, since, according to Fig. 4, $|O_i^{Z'}(\tau) - O_i^{Z'}(\mu)| = |O_i^{Z'}(\tau) - O_i^{\mathrm{SM}}(\tau)| + |O_i^{Z'}(\mu) - O_i^{\mathrm{SM}}(\mu)|$, and therefore $[O_i^{Z'}(\tau) - O_i^{Z'}(\mu)]^2 > \{[O_i^{Z'}(\tau) - O_i^{\mathrm{SM}}(\tau)]^2 + [O_i^{Z'}(\mu) - O_i^{\mathrm{SM}}(\mu)]^2\}$, and $\delta O_i^{Z'}(\mu) \simeq \delta O_i^{\mathrm{SM}}(\mu)$, (compare with (17) and (18)). Therefore, in the presence of the SM, the μ–τ distinguishability limit of the $\mathrm{Sp}(6)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ model, is forced down to the Z' discovery limit, $(M_{\mu-\tau})_{\mathrm{Sp}} = (M_\mathrm{d})_{\mathrm{Sp}}$. This means that the Z' discovery limit defines also a limit up to which the deviation of the observable from the SM value depends on the generation. We have seen the important role that the τ observables play in the differentiation of the $\mathrm{Sp}(6)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ model from other models, especially model I'. Now, as model I' comes in the picture, a limit can be defined for the gauge boson mass M, up to which model I' can be differentiated from the $\mathrm{Sp}(6)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ model. Having $(M_{\mu-\tau})_{\mathrm{Sp}} = (M_\mathrm{d})_{\mathrm{Sp}}$, this limit should be equal to the smaller one of the discovery limits of Z' and $Z_{I'}$. Figure 6 shows that, at $\sqrt{s} = 0.5$ TeV and $L = 0.5\ \mathrm{ab}^{-1}$, and based on the observables considered in this analysis, the 99% C.L. discovery limit for $Z_{I'}$ is $(M_\mathrm{d})_{I'} \sim 5.2$ TeV. Since $(M_\mathrm{d})_{I'} < (M_{\mu-\tau})_{\mathrm{Sp}}$, it will therefore be possible, at this collision energy, to differentiate model I' from the $\mathrm{Sp}(6)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ model up to gauge boson masses $M \lesssim 5.2$ TeV. Beyond this limit, and up to the Z' discovery limit, only the $\mathrm{Sp}(6)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ model can be identified.

In the above calculations we considered only statistical errors. Because the bounds obtained at e^+e^- colliders are indirect, based on deviations from the SM, they are sensitive to experimental errors, both statistical and systematic. We calculated the total error by combining in quadrature the statistical error and the systematic error. For example, including a 0.5% (2%) systematic error in the cross sections due to, for example, luminosity uncertainties, and a 0.25% (1%) one in the asymmetries, where systematic errors partially cancel, reduces the discovery limits by 17% (28%) for $\sqrt{s} = 1$ TeV and $L = 1\ \mathrm{ab}^{-1}$. Thus, including even a small systematic error reduces the limits substantially. Systematic errors will therefore have to be kept under control. Another factor that affects the limits is tau tagging. We point out here that the limits are expected to be reduced further when the tau tagging efficiencies are included into the calculations.

In conclusion, the $\mathrm{Sp}(6)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ extension of the standard model gauge group suggests an additional neutral gauge boson, Z', that couples equally to the first two generations of fermions but differently to the third. We studied distributions of leptons in high-energy e^+e^- collisions, produced by Z' as well as by other theoretically motivated neutral gauge bosons. In particular, we considered the gauge bosons Z_{LR}, Z_χ, Z_ψ, Z_η and $Z_{I'}$ occurring in left–right symmetric models and in models based on the E_6 and SO(10) groups. Because of the generation-dependent couplings of fermions to Z', a four-fold enhancement is expected in its on-resonance tau-pair production rate relative to its muon-pair production rate. Because of coupling considerations of leptons to the extra gauge boson, lepton-pair production rates from the $\mathrm{Sp}(6)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ model are expected to be confused with those from model I' and model χ. Moreover, the $\mathrm{Sp}(6)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ model cannot be distinguished from model I' through only measurements of the forward–backward asymmetry, $A_{\mathrm{FB}}(\mu)$, or the left–right asymmetry, $A^P_{\mathrm{LR}}(\mu)$, of the muon-pair final state. However the $\mathrm{Sp}(6)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ model predictions for the tau-pair asymmetries, $A_{\mathrm{FB}}(\tau)$ and $A^P_{\mathrm{LR}}(\tau)$, are well separated from the predictions of any of the other models and can thereby be used to easily distinguish the model. We studied the discovery potential of Z' at various planned and proposed e^+e^- colliders. We found that the universality violation inherent in the $\mathrm{Sp}(6)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ model not only played an important role in the differentiation of the model from other models but also enhanced the discovery limits for Z'.

References

1. S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967)
2. A. Salam, in Elementary Particle Theory: Relativistic Groups and Analyticity (Noble Symposium No. 8), edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367
3. P. Langacker, M. Plümacher, Phys. Rev. D **62**, 013 006 (2000)
4. J. Erler, P. Langacker, Phys. Rev. Lett. **48**, 212 (2000)
5. T.K. Kuo, N. Nakagawa, Phys. Rev. D **30**, 2011 (1984)
6. T.K. Kuo, N. Nakagawa, Nucl. Phys. B **250**, 641 (1985)
7. A.A. Bagneid, T.K. Kuo, N. Nakagawa, Int. J. Mod. Phys. A **2**, 1351 (1987)
8. V. Barger et al., Int. J. Mod. Phys. A **2**, 1327 (1987)
9. T.K. Kuo, N. Nakagawa, Phys. Rev. D **31**, 1161 (1985)
10. T.K. Kuo, N. Nakagawa, Phys. Rev. D **32**, 306 (1985)
11. G.T. Park, T.K. Kuo, Phys. Rev. D **42**, 3879 (1990)
12. G.T. Park, T.K. Kuo, Phys. Rev. D **45**, 1720 (1992)
13. G.T. Park, T.K. Kuo, Z. Phys. C **59**, 445 (1993)
14. A.A. Bagneid, T.K. Kuo, G.T. Park, Phys. Rev. D **44**, 2188 (1991)
15. T.G. Rizzo, Phys. Rev. D **42**, 3755 (1990)
16. J.L. Hewett, T.G. Rizzo, Phys. Rev. D **45**, 161 (1992)
17. T.G. Rizzo, Phys. Rev. D **46**, 3751 (1992)
18. A.A. Bagneid, Phys. Rev. D **49**, 3755 (1994)
19. T.K. Kuo, G.T. Park, Phys. Rev. D **50**, 3508 (1994)
20. T.K. Kuo, G.T. Park, M. Zarlek, Nuovo Cim. A **107**, 757 (1994)
21. A.A. Bagneid, Nuovo Cim. A **110**, 747 (1997)
22. A.A. Bagneid, Nuovo Cim. A **112**, 761 (1999)
23. A.A. Bagneid, Eur. Phys. J. C **22**, 743 (2002)
24. A.A. Bagneid, Int. J. Mod. Phys. A **17**, 1273 (2002)
25. A.A. Bagneid, Phys. Rev. D **66**, 017 701 (2002)
26. A.A. Bagneid, Int. J. Mod. Phys. A **20**, 7255 (2005)
27. P. Langacker, R.W. Robinett, J.L. Rosner, Phys. Rev. D **30**, 1470 (1984)
28. D. London, J.L. Rosner, Phys. Rev. D **34**, 1530 (1986)
29. J.L. Hewett, T.G. Rizzo, Phys. Rep. **183**, 193 (1989)
30. P. Langacker, M. Luo, Rev. Mod. Phys. **64**, 87 (1992)
31. J.C. Pati, A. Salam, Phys. Rev. D **10**, 275 (1974)
32. V. Barger, E. Ma, K. Whisnant, Phys. Rev. D **28**, 1618 (1983)
33. P. Langacker, R.W. Robinett, J. Rosner, Phys. Rev. D **30**, 1470 (1984)
34. L.S. Durkin, P. Langacker, Phys. Lett. B **166**, 436 (1986)
35. U. Amaldi et al., Phys. Rev. D **36**, 1385 (1987)
36. G. Costa et al., Nucl. Phys. B **297**, 244 (1988)
37. A.A. Bagneid, T.K. Kuo, Phys. Rev. D **35**, R2275 (1987)
38. A.A. Bagneid, T.K. Kuo, Phys. Rev. D **38**, 2153 (1988)
39. T.G. Rizzo, Phys. Rev. D **34**, 2699 (1986)
40. J.L. Hewett, T.G. Rizzo, Phys. Rep. **183**, 193 (1989)
41. A.A. Bagneid, T.K. Kuo, G.T. Park, Phys. Rev. D **44**, 2188 (1991)
42. F. del Aguila et al., Nucl. Phys. B **361**, 45 (1991)
43. F. del Aguila et al., Nucl. Phys. B **372**, 3 (1992)
44. P. Langacker, M. Luo, Phys. Rev. D **45**, 278 (1992)
45. P. Langacker, M. Luo, Rev. Mod. Phys. **64**, 87 (1992)
46. G. Altarelli et al., Nucl. Phys. B **405**, 3 (1993)
47. A.A. Bagneid, Phys. Rev. D **49**, 3755 (1994)
48. F. Abe et al., Phys. Rev. D **51**, R949 (1995)
49. S. Godfrey, Mod. Phys. Lett. **12**, 1859 (1997)
50. T.G. Rizzo, Phys. Rev. D **61**, 055 005 (2000)
51. S. Godfrey et al., Phys. Rev. D **61**, 113 009 (2000)
52. R. Barate et al., Eur. Phys. J. C **12**, 183 (2000)
53. P. Langacker, M. Plümacher, Phys. Rev. D **62**, 013 006 (2000)
54. G. Abbiendi et al., Eur. Phys. J. C **33**, 173 (2004)
55. F. Abe et al., Phys. Rev. Lett. **79**, 2192 (1997)
56. V. Barger et al., Int. J. Mod. Phys. A **2**, 1327 (1987)
57. J.L. Hewett, T.G. Rizzo, Phys. Rev. D **45**, 161 (1992)
58. J.L. Hewett, T.G. Rizzo, Phys. Rev. D **47**, 4981 (1993)
59. A.A. Bagneid, Eur. Phys. J. C **22**, 743 (2002)
60. A.A. Bagneid, Phys. Rev. D **66**, 017 701 (2002)
61. http://www.linearcollider.org/cms/
62. CLIC Physics Group, E. Accomando et al., arXiv:hep-ph/0412251
63. A. Leike, Phys. Rep. **317**, 143 (1999)
64. J. Erler, P. Langacker, Phys. Lett. B **456**, 68 (1999)
65. R. Barate et al., Eur. Phys. J. C **12**, 183 (2000)
66. T.K. Kuo, G.T. Park, Phys. Rev. D **50**, 3508 (1994)
67. G.T. Park, T.K. Kuo, Int. J. Mod. Phys. A **10**, 4387 (1995)
68. P. Langacker, M. Luo, Phys. Rev. D **45**, 278 (1992) [In calculating Γ_2 we assumed that 27-plet channels are open for $Z\psi$, $Z\eta$ and $Z_{I'}$ and three 16-plets for Z_χ and Z_LR.]
69. F. Abe et al., Phys. Rev. Lett. **79**, 2192 (1997)
70. J. Erler, P. Langacker, Phys. Lett. B **456**, 68 (1999)
71. R. Barate et al., Eur. Phys. J. C **12**, 183 (2000)
72. G. Abbiendi et al., Eur. Phys. J. C **33**, 173 (2004)
73. A.A. Bagneid, Int. J. Mod. Phys. A **17**, 1273 (2002)
74. LHC/LC Study group, G. Weiglein et al., arXiv:hep-ph/0410364
75. CLIC Physics Group, E. Accomando et al., arXiv:hep-ph/0412251
76. G. Moortgat-Pick et al., arXiv:hep-ph/0507011
77. S. Godfrey, Phys. Rev. D **51**, 1402 (1995)
78. S. Godfrey, in Proc. of 1996 PDF/DPB Summer Study on New Directions for High Energy Physics (Snowmass 96) arXiv:hep-ph/9612384
79. M. Cvetic, S. Godfrey, in Electro-weak symmetry Breaking and Beyond the Standard Model, ed. by T. Barklow, S. Dawson, H. Haber, J. Seigrist (World Scientific) arXiv:hep-ph/9504216
80. A. Leik, S. Riemann, in Proc. of Physics with e^+e^- Linear Colliders Workshop (Annecy-Gran Sasso-, Hamburg, 1995) ed. by P. Zerwas, arXiv:hep-ph/9604321
81. A. Leike, talk presented at the symposium 100 years Werner Heisenberg – Works and Impact, arXiv:hep-ph/0111300
82. LHC/LC Study group, G. Weiglein et al., arXiv:hep-ph/0410364
83. S. Godfrey, in International Linear Collider Workshop (Stanford, U.S.A.), arXiv:hep-ph/0508182
84. S. Gofrey P. Klyniak, A. Tomkins, in 2005 ALCPG & ILC Workshop (Snowmass, U.S.A.), arXiv:hep-ph/0511335